# Slow-Roll Suppression of Adiabatic Instabilities in Coupled Scalar Field-Dark Matter Models

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We study the evolution of linear density perturbations in the context of interacting scalar field-dark matter cosmologies, where the presence of the coupling acts as a stabilization mechanism for the runaway behavior of the scalar self-interaction potential as in the case of the Chameleon model. We show that in the "adiabatic" background regime of the system the rise of unstable growing modes of the perturbations is suppressed by the slow-roll dynamics of the field. Furthermore the coupled system behaves as an inhomogeneous adiabatic fluid. In contrast instabilities may develop for large values of the coupling constant, or along non-adiabatic solutions, characterized by a period of high-frequency dumped oscillations of the scalar field. In the latter case the dynamical instabilities of the field fluctuations, which are typical of oscillatory scalar field regimes, are amplified and transmitted by the coupling to dark matter perturbations.

## I. INTRODUCTION

Cosmology has provided evidence of a dark physics sector which is necessary to account for about 95% of the cosmic matter content [1]. Despite the success of the ΛCDM model to fit all cosmological observations, the existence of the dark energy phenomenon as well as its relation to the abundance and clustering of matter in the universe still pose puzzling questions.

Models of interacting dark energy-dark matter have been proposed to address such problems. In this scenario dark energy is a fundamental scalar field which directly couples to matter particles. This allows for a dynamical solution of the so called "coincidence" problem, since independently of the initial conditions the scalar interaction drives the dark energy-to-matter ratio toward a constant value (see e.g. [2, 3, 4, 5]). These models are inspired by string and supergravity theories, where the compactification of extra-dimensions in the low-energy gives rise to massless scalars coupled to matter fields with gravitational strength. Therefore a distinct feature of this scenario is that matter particles experience a long-range scalar force and acquire a time dependent mass which cause violations of the Equivalence Principle (EP). The tight bounds imposed by EP tests are usually avoided as consequence of other possible mechanisms. As an example Damour and Polyakov have shown that in String Theory the couplings between the dilaton and different matter fields can be dynamically suppressed [6]. An interesting possibility has been proposed in the "Chameleon" model [7], where the mass of the scalar field is assumed to depend on the local matter density. In such a case fifth-force effects can be strongly suppressed on Solar System scales, thus avoiding EP bounds. Another possibility has been explored in [8] where the authors consider a dilatonic field to be differently coupled to various matter species such that the system can naturally evolve toward a late time attractor solution where

General Relativity is recovered. Non-minimally coupled models can successfully describe the background expansion of the universe as probed by supernova type Ia luminosity distance or the position of the Doppler peaks in the Cosmic Microwave Background anisotropy power spectrum (see e.g. [9, 10]). However testing the formation of structure in the universe more than standard cosmological tests may provide a key insight on this class of models. In fact the scalar coupling contributes to modifying the clustering properties of matter, implying that an accurate study of the evolution of density fluctuations both in the linear and non-linear phase of collapse can identifying unique signatures of dark sector interactions [11]. In the context of linear perturbation theory several interacting scalar field-dark matter models have been studied in the literature (see e.g. [12]). In some specific realizations it was found that the growth of linear density perturbations is spoiled by the presence of dangerous instabilities [13, 14], as in the case of "Mass Varying Neutrino" (MaVaN) models [15]. Recently a number of works have analysed the stability of perturbations in more general setups. For instance in [16] the authors have studied models with a background evolution characterized by an adiabatic regime, and shown that unstable growing modes of the perturbations exist for couplings much greater than gravitational strength. On the other hand the authors of [17, 18] have considered the case of an interacting dark energy component with constant equation of state and found that for couplings proportional to the dark matter density the perturbations are unstable.

In this paper we provide a more detailed study of these instabilities, particularly in relation to the specificities of the background scalar field evolution. The paper is organized as follows: in Section II we introduce the interacting scalar field-matter model as well as the background and perturbation equations; in Section III we present the results of our analysis; finally in Section IV we present our conclusions.

## II. INTERACTING SCALAR FIELD-DARK MATTER MODEL

Let us consider a scalar field $\phi$ with direct coupling to matter particles via a Yukawa term $f(\phi/M_{Pl})\bar{\psi}\psi$, where $f$ is the coupling function and $\psi$ is a Dirac spinor representing the matter field ($M_{Pl} = 1/\sqrt{8\pi G}$ is the reduced Planck mass with G being the Newton constant). The effect of the scalar-dependent coupling is to induce a time-varying mass of the matter particles, hence causing a violation of the EP. As mentioned in the previous Section, there are several ways to evade the tight bounds from EP tests. Here we assume that the scalar field only couples to dark matter particles. Therefore for the purposes of our analysis we neglect the baryon contribution and focus only on the cosmological evolution of the coupled scalar field-dark matter system.

As in the case of the Chameleon cosmology [19], we assume the $\phi$-field to have a self-interaction potential of runaway type in the form of inverse power-law:

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha}, \tag{1}$$

where $M$ is a mass scale and $\alpha$ is a positive constant. We consider a coupling function of dilatonic type, $f(\phi) = \exp(\beta\phi/M_{Pl})$, with $\beta$ a dimensionless coupling constant. The background evolution of this system has been studied in detail in [9].

### A. Background and Linear Perturbation Equations

Let assume a flat Friedmann-Lemaitre-Robertson-Walker metric ($ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$), the evolution of the scale factor is given by:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left[\rho_{DM} + \dot{\phi}^2/2 + V(\phi)\right], \tag{2}$$

where $\rho_{DM}$ is the dark matter density and we have adopted Planck units ($M_{Pl} = 1$). The total energy momentum tensor of the system is conserved, $T_{\nu;\mu}^{\mu(T)} \equiv T_{\nu;\mu}^{\mu(DM)} + T_{\nu;\mu}^{\mu(\phi)} = 0$. In contrast the non-minimal coupling implies that the energy momentum tensor of each individual component is not conserved. In such a case we can consider

$$T_{\nu;\mu}^{\mu(DM)} = \beta\phi_{;\nu}T_\gamma^{\gamma(DM)}, \tag{3}$$
$$T_{\nu;\mu}^{\mu(\phi)} = -\beta\phi_{;\nu}T_\gamma^{\gamma(DM)}, \tag{4}$$

from which we obtain:

$$\dot{\rho}_{DM} + 3H\rho_{DM} = \beta\dot{\phi}\rho_{DM}, \tag{5}$$
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\beta\rho_{DM}. \tag{6}$$

Without loss of generality we can rescale the coupling function $f(\phi)$ to its present value, $f(\phi_0)$. Hence the solution to Eq. (5) is
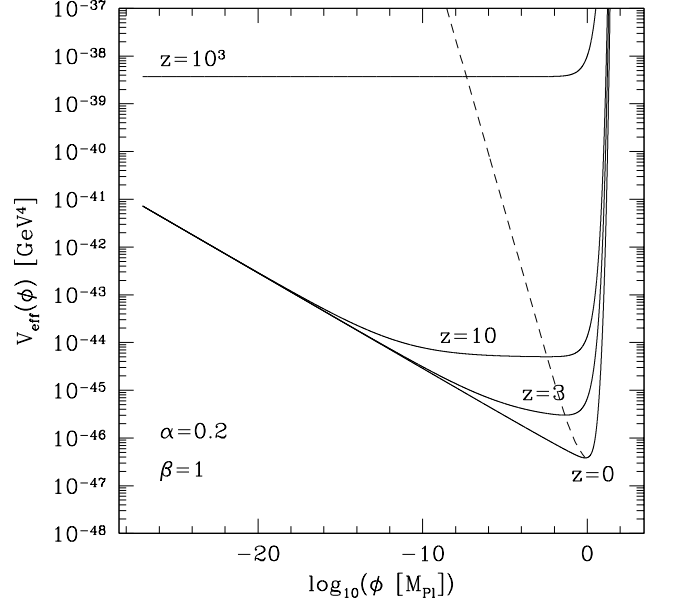


FIG. 1: Scalar field effective potential at $z = 10^3, 10, 3$ and $0$ (solid lines) for $\alpha = 0.2$ and $\beta = 1$. The amplitude of the scalar potential $M$ is set such that today $\Omega_{DM} = 0.24$ ($\Omega_\phi = 1 - \Omega_{DM}$). The dashed line corresponds to the position of the minimum of the effective potential at different epochs.

$$\rho_{DM} = \frac{\rho_{DM}^{(0)}}{a^3}e^{\beta(\phi-\phi_0)}, \tag{7}$$

where $\rho_{DM}^{(0)}$ is the present matter density. We may notice that as consequence of the scalar interaction the dark matter density deviates from the standard scaling $a^{-3}$. Furthermore for coupling values $\beta > 0$, the system of Eqs. (5)-(6) describes an energy transfer from the $\phi$-field to dark matter. In such a case the scalar field evolves in an effective potential

$$V_{\rm eff}(\phi) = V(\phi) + \frac{\rho_{DM}^{(0)}}{a^3}e^{\beta(\phi-\phi_0)}, \tag{8}$$

which is characterized by the presence of a minimum.

In figure 1 we plot the effective potential for $\beta = 1$ and $\alpha = 0.2$ at redshift $z = 1000, 10, 3$ and $0$ respectively. For this choice of the model parameters we have $\phi_0 \approx 0.7605$ as obtained by integrating numerically the system of Eqs. (5)-(2). The dashed line in Fig. 1 corresponds to the position of the minimum at different epochs.

In synchronous gauge the linearized equation for the dark matter density contrast $\delta_{DM}$, velocity gradient $\theta_{DM}$, and field fluctuation $\delta\phi$ are given by:

$$\dot{\delta}_{DM} = -\left(\frac{\theta_{DM}}{a} + \frac{\dot{h}}{2}\right) + \beta\dot{\delta\phi}, \tag{9}$$

$$\dot{\theta}_{DM} = -H\theta_{DM} + \beta\left(\frac{k^2}{a}\delta\phi - \dot{\phi}\theta_{DM}\right), \qquad (10)$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V_{,\phi\phi}\right)\delta\phi + \frac{1}{2}\dot{h}\dot{\phi} =$$
$$= -\beta\rho_{DM}\delta_{DM}, \qquad (11)$$

where $h$ is the metric perturbation given by:

$$\dot{h} = \frac{2k^2\eta}{a^2H} - \frac{8\pi G}{H}\left[\delta\rho_\phi + \rho_{DM}\delta_{DM}\right], \qquad (12)$$

with $\delta\rho_\phi = \dot{\phi}\delta\dot{\phi} + V_{,\phi}\delta\phi$ and

$$\dot{\eta} = \frac{4\pi G}{k^2}a\left[\rho_{DM}\theta_{DM} + ak^2\dot{\phi}\delta\phi\right]. \qquad (13)$$

In Section III we will present the results of the numerical integration of this system of equations. However for a qualitative understanding of the conditions which lead to the onset of instabilities during the growth of the density perturbations, it is useful to introduce an effective unified fluid description.

## B. Effective Unified Fluid Description

The conservation of the total energy momentum tensor allows us to describe the interacting scalar field-dark matter system as a single unified fluid. The equation for the background density is given by

$$\dot{\rho}_T = -3H(1 + w_T)\rho_T, \qquad (14)$$

with $\rho_T = \dot{\phi}^2/2 + V(\phi) + \rho_{DM}$ and $w_T = p_T/\rho_T$, where $p_T = \dot{\phi}^2/2 + V(\phi)$. Similarly at linear order the perturbation equations in synchronous gauge read as:

$$\dot{\delta}_T = -3H(c_{sT}^2 - w_T)\delta_T +$$
$$- (1 + w_T)\left\{\left[\frac{k^2}{a^2H^2} + 9(c_{sT}^2 - c_{aT}^2)\right]\frac{aH^2}{k^2}\theta_T + \frac{\dot{h}}{2}\right\}, \qquad (15)$$

$$\dot{\theta}_T = -H(1 - 3c_{sT}^2)\theta_T + \frac{c_{sT}^2 k^2}{a(1 + w_T)}\delta_T, \qquad (16)$$

where $c_{aT}^2 = \dot{p}_T/\dot{\rho}_T$ is the square of the adiabatic sound speed of the unified fluid and $c_{sT}^2 = \delta p_T/\delta\rho_T$ is the square of the speed at which pressure perturbations propagate in the fluid rest frame. For a barotropic fluid with a constant equation of state (e.g. matter, radiation) $c_s^2 = c_a^2 = w$. This is not the case for a generic fluid (e.g. scalar field), for this reason we may expect the effective unified fluid to be non-barotropic, (i.e. $c_{sT}^2 \neq c_{aT}^2 \neq w_T$). In terms of the scalar field and dark matter variables we have

$$c_{aT}^2 = \frac{3H\dot{\phi}^2 + \dot{\phi}[2V_{,\phi} + \beta\rho_{DM}]}{3H\dot{\phi}^2 + 3H\rho_{DM}}, \qquad (17)$$

$$c_{sT}^2 = \frac{\dot{\phi}\delta\dot{\phi} - V_{,\phi}\delta\phi}{\dot{\phi}\delta\dot{\phi} + V_{,\phi}\delta\phi + \rho_{DM}\delta_{DM}}. \qquad (18)$$

These relations provide us with a simple way of determining the properties of the perturbation in the coupled system. For example in a given background regime instabilities of the perturbations may develop if the adiabatic sound speed acquire sufficiently negative values.

## III. SCALAR FIELD DYNAMICS AND EVOLUTION OF DENSITY PERTURBATIONS

The non-minimally coupled scalar field model described in Section II is characterized by the existence of an attractor solution which is set by the minimum of the effective potential. The minimum is given by $V_{\text{eff}}^{,\phi} = 0$, thus along the attractor solution the following condition is always satisfied:

$$V_{,\phi} = -\beta\frac{\rho_{DM}^{(0)}}{a^3}e^{\beta(\phi-\phi_0)}. \qquad (19)$$

Evaluating the derivative of Eq. (1) and substituting in Eq. (19) we obtain the time evolution of the field at the minimum:

$$\left(\frac{\phi_0}{\phi_{\min}}\right)^{\alpha+1} = \frac{1}{a^3}e^{\beta(\phi_{\min}-\phi_0)}, \qquad (20)$$

which depends on both the slope $\alpha$ and the coupling $\beta$. Equation (20) is a non-linear algebraic equation which can be solved numerically through standard bisection methods (see dashed line in Fig. 1).

The field may reach the minimum from two different sets of initial conditions: $\phi_{ini} < \phi_{\min}^{ini}$ (small field) or $\phi_{ini} > \phi_{\min}^{ini}$ (large field). In the former case $\phi$ evolves over the inverse power-law part of the effective potential, where it minimizes the potential by slow-rolling as shown in [9]. In fact one can easily verify that throughout the cosmological evolution the field mass ($m^2 = V_{\text{eff}}^{,\phi\phi}$) as well as the ratio of its kinetic-to-potential energy satisfy the conditions $m > H$ and $\dot{\phi}^2/2V < 1$ respectively. In contrast starting from large field values, $\phi$ rolls towards the minimum along the steep exponential part of $V_{\text{eff}}(\phi)$. Thus it rapidly acquires kinetic energy which subsequently dissipates through large high-frequency damped oscillations around the minimum.

As we shall see next, the growth of linear perturbations in these two regimes is significantly different.

## A. Adiabatic Regime

As mentioned in Section II B we can obtain a qualitative insight on the stability of the perturbations in the coupled system by considering the effective unified fluid description. Let us evaluate the adiabatic sound speed Eq. (17) along the adiabatic solution Eq. (19); after neglecting the term proportional to the kinetic energy of the scalar field we have

$$c_{aT}^2 = -\beta\frac{\dot{\phi}}{3H}, \qquad (21)$$

since $\dot{\phi} > 0$ it then follows that $c_{aT}^2 < 0$, implying that adiabatic instabilities may indeed develop. However we should remark that during the adiabatic regime the field is slow-rolling (i.e. $3H\dot{\phi} \approx 0$), hence the term $\dot{\phi}/3H$ can be negligibly small compared to $\beta$, such that $c_{aT}^2 \approx 0$, hence leading to a stable growth of the perturbations. In contrast instabilities will occur if the coupling assumes extremely large values, $\beta \gg 3H/\dot{\phi}$. This is consistent with the analysis presented in [16], where the authors have suggested that during the adiabatic regime perturbations suffer of instabilities provided that $\beta \gg 1$. Here we want to stress two main points which were not addressed in that study: first of all that the rise of instabilities is suppressed by the slow-rolling of the field in the adiabatic regime, and secondly that exactly because of the slow-roll condition, instabilities can spoil the growth of dark matter perturbations only for large unnatural values of the coupling. To give an example let us assume that for a given model along the adiabatic solution the following condition occurs: $\dot{\phi}/3H \sim 10^{-2}$. In such a case instabilities will develop only if the coupling constant $\beta > 100$, corresponding to a scalar fifth-force which is 2000 times greater than the gravitational strength. [1]

Moreover during the adiabatic evolution, Eq. (18) reads as

$$c_{sT}^2 = -\frac{1}{1 - \frac{1}{\beta}\frac{\delta_{DM}}{\delta\phi}}, \qquad (22)$$

and assuming that the scalar field is nearly homogeneous, $\delta\phi \ll \delta_{DM}$ (in Planck units), we have $c_{sT}^2 \approx \beta\delta\phi/\delta_{DM}$, and for $\beta \approx \mathcal{O}(1)$ this implies $c_{sT}^2 \approx 0$. In other words if the scalar field fluctuations are small with respect to the dark matter density contrast, then the coupled system behaves has a single adiabatic inhomogeneous fluid ($c_{sT}^2 \approx c_{aT}^2 \approx 0$).

These results are supported by the numerical study of the system of Eqs. (9)-(13), with the scalar field evolution given by Eq. (20). We have set the model parameters to the the following values: $\alpha = 0.2$, $\beta = 1$, with $\Omega_{DM} = 0.24$, $H_0 = 70\ \mathrm{Km}\,s^{-1}\mathrm{Mpc}^{-1}$. As shown in [9] this model has the interesting feature that the background dynamics can mimic that of a phantom cosmology corresponding to an uncoupled dark energy model with slightly constant super-negative equation of state $w_{DE} = -1.1$.

The results of the numerical integration are shown in Figure 2. In the upper left panel we plot the scalar field equation of state $w_\phi$ (solid line) and the equation of state for the effective unified fluid $w_T$ (dot line). As we can see $w_\phi = -1$, which is consistent with the fact that $\dot{\phi}/3H$ is negligible, as can be seen from the plot in the right
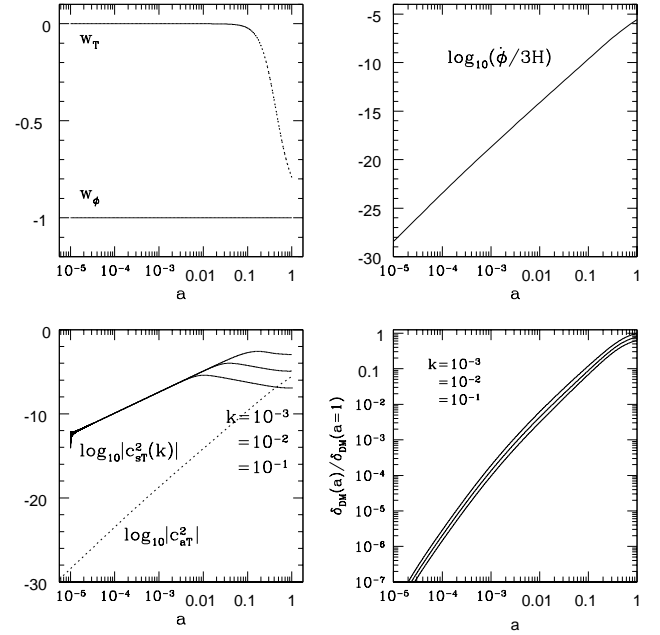


FIG. 2: Upper left panel: evolution of the scalar field equation of state $w_\phi$ and effective unified fluid equation of state $w_T$; Right upper panel: evolution of the scalar field velocity with respect to the Hubble rate; Lower left panel: redshift evolution of the adiabatic sound speed $c_{aT}^2$ and propagation of pressure perturbations $c_{sT}^2$; Right lower panel: Linear growth factor of the dark matter density contrast at $k = 10^{-3}, 10^{-2}$ and 0.1 Mpc$^{-1}$ .

upper panel. We can also notice that the unified fluid at early times behaves as a matter component ($w_T = 0$) and deviates toward negative values ($-1 < w_T < 0$) as the $\phi$-field becomes the energetically dominant. In the lower left panel we plot the absolute value of $c_{aT}^2$, and $c_{sT}^2(k)$ for three different scales $k = 10^{-3}, 10^{-2}$ and 0.1 Mpc$^{-1}$ respectively. The adiabatic sound speed has negligible negative values and evolves with a trend that matches that of $\dot{\phi}/3H$, which is consistent with Eq. (21). We can also notice that the speed of propagation of pressure perturbations in the unified fluid remains is $\approx 0$. Hence during the adiabatic regime the interacting system behaves as a single inhomogeneous adiabatic fluid. In the lower right panel we plot the evolution of the dark matter density contrast normalized to the present value for $k = 10^{-3}, 10^{-2}$ and 0.1 Mpc$^{-1}$ respectively (for clarity we have displaced by a constant factor the different curves which would otherwise nearly overlap). As expected these different modes manifest a standard power law growth and no instabilities are present. These results have been obtained for an inverse power-law potential, nevertheless they can be generalized to other scalar potentials, the only requirement is the existence of an adiabatic solution during which the slow-roll condition is satisfied.

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[1] As consequence of the scalar interaction dark matter particles experience a gravitational force with effective Newtonian constant $G_{\mathrm{eff}} = G(1 + 2\beta^2)$. In contrast baryonic bodies may not experience such modification due to the non-linear nature of the scalar interaction [20].

## B.  Non-Adiabatic Regime: Large Field Oscillations

Starting from initially large field values, the system evolves along a non-adiabatic solution characterized by rapid dumped field oscillations around the minimum of the effective potential. We can see this explicitely the upper left panel of Figure 3, where we plot the evolution of the scalar field equation of state for the same model parameters as in Section III A and obtained by numerically integrating Eq. (6) with initial conditions: $\phi(a_{ini} = 10^{-5}) = 0.15 > \phi_{\min}^{ini}$ and $\dot{\phi}_{ini} = 0$. We can infer the main features of the scalar field evolution from the behavior of it equation of state shown in the upper left panel of Figure 3. As we can see the field initially behaves as a stiff component with $w_\phi = 1$, this is because the field starts rolling on the steep exponential part of the potential, and consequently its energy is dominated by the kinetic term. As the field reaches the opposite side of the potential, it undergoes a series of high-frequency dumped oscillations around the minimum during which it dissipates most of its kinetic energy. It then sets on the inverse power-law part of the potential where it evolves along a tracker solution with $w_\phi \approx -2/(2+\alpha) \approx -0.9$.

The evolution of density perturbations in the case of oscillating scalar fields has been widely studied in the literature, particularly in context of inflation [21]. From these studies it is well known that scalar field fluctuations are unstable during oscillatory regimes. In [22] the authors have presented a simple insightful explanation for the onset of such instabilities. The idea is to interpret the scalar fluctuation $\delta\phi$ as the separation between two particles whose dynamics is described by two coupled anharmonic oscillators. Then a simple stability criterion is given by the relation between the frequency of the oscillations $\omega$, and their amplitude $\tilde{\phi}$ [23]. Let us suppose that the frequency increases as the amplitude of the oscillations diminishes, in such a case it has been shown that the distance between the two particles increases, thus causing the scalar field fluctuation to be unstable [22]. This is indeed what occurs in the interacting scalar field-dark matter system along the non-adiabatic solution we are considering. In fact we can see in the right upper panel of Fig. 3 that as the field starts oscillating, the frequency of the oscillations increases as the field amplitude diminishes, $(d\omega/d\tilde{\phi} < 0)$. We can therefore expect the presence of instable modes. This is confirmed by the numerical solutions of $\delta\phi_k$ and $\delta_{DM}$ obtained from the integration of Eqs. (9)-(13). The evolution of the scalar field fluctuation $\delta\phi_k$ is shown in the lower left panel of Fig. 3. We may notice the presence of an instability occurring roughly at the same time of the first oscillation, then followed by a second stage of exponential growth at the beginning of the second oscillation. From the plot in the lower right panel we can also see that the same instability is passed to the dark matter perturbation, which is a direct consequence of the coupling terms in Eq. (9) and Eq. (10). Such unstable modes are similar to those found in [17, 18], in fact by averaging over periods of time larger
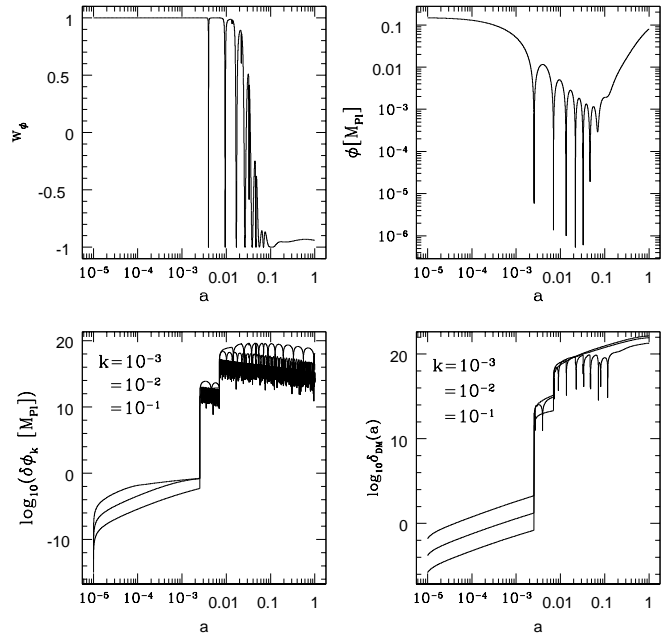


FIG. 3: Upper left panel: evolution of the scalar field equation of state $w_\phi$; Right upper panel: evolution of the scalar field; Lower left panel: evolution of the field fluctuations $\delta\phi_k$ at $k = 10^{-3}, 10^{-2}$ and 0.1 Mpc$^{-1}$ respectively; Right lower panel: evolution of dark matter density for $k$-values as in the case of $\delta\phi_k$.

than the characteristic time of the oscillations, the scalar field behaves effectively as a dark energy component with a constant equation of state.

## IV.  CONCLUSIONS

We have studied the evolution of linear perturbations in the case of an interacting scalar field with runaway potential directly coupled to dark matter particles. We have specifically analyzed the stability of perturbations during the adiabatic evolution of the field, and shown that as consequence of the slow-roll condition the onsets of instabilities is largely suppressed. This can be explained in terms of the adiabatic sound speed of the effective unified fluid. In fact during the adiabatic regime, despite being negative, it assumes negligibly small values and as consequence of this the growth of linear density perturbations remains stable. On the other hand instabilities may develop in strongly coupled adiabatic regimes, with a coupling constant much greater than gravitational strength. Interestingly during the adiabatic evolution of the field the coupled system behaves as a single adiabatic inhomogeneous fluid. We have also shown that large instabilities can spoil the growth of linear perturbations in the case of non-adiabatic solutions characterized by large scalar field oscillations. It is well known that scalar field

fluctuations are unstable during oscillatory regimes, in such a case the scalar coupling amplifies and propagates such instabilities to the perturbations of the dark matter component.

Our analysis suggest that under minimal natural model assumptions Chameleon-like cosmologies are not affected by instabilities of the perturbations and can provide a viable period of structure formation more than previously believed.

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